Logic Gates and Boolean Functions

and first steps with CircuitVerse

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Boolean Function (Scalar Valued)

- Function of the form

$$f : B^n \to B$$

with

$$B := \{0, 1\}$$

- We can interpret “0 as false”, “1 as true”

NOT Gate

Function

$$f : B \to B, \; x \mapsto f(x)$$

with

$$f(x) = 1 - x$$

NOT Gate

Function

$$\neg : B \to B,\ x \mapsto \neg x$$

with

$$\neg x = 1 - x$$

NOT Gate

Function

$$\neg : B \to B,\ x \mapsto \neg x$$

with

$$\neg x = 1 - x$$

x	$\neg x$

NOT Gate

Function

$$\neg : B \to B,\ x \mapsto \neg x$$

with

$$\neg x = 1 - x$$

x	$\neg x$
0	1

NOT Gate

Function

$$\neg : B \to B,\ x \mapsto \neg x$$

with

$$\neg x = 1 - x$$

x	$\neg x$
0	1
1	0

AND Gate

Function

$$\wedge : B^2 \to B, (a, b) \mapsto a \wedge b$$

with

AND Gate

Function

$$\wedge : B^2 \to B, (a, b) \mapsto a \wedge b$$

with

a	b	$a \wedge b$

AND Gate

Function

$$\wedge : B^2 \to B, (a, b) \mapsto a \wedge b$$

with

a	b	$a \wedge b$
0	0	0

AND Gate

Function

$$\wedge : B^2 \to B, (a, b) \mapsto a \wedge b$$

with

a	b	$a \wedge b$
0	0	0
0	1	0

AND Gate

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$$\wedge : B^2 \to B, (a, b) \mapsto a \wedge b$$

with

a	b	$a \wedge b$
0	0	0
0	1	0
1	0	0

AND Gate

Function

$$\wedge : B^2 \to B, (a, b) \mapsto a \wedge b$$

with

a	b	$a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate

Function

$$\vee : B^2 \to B, \ (a, b) \mapsto a \vee b$$

with

OR Gate

Function

$$\vee : B^2 \to B,\ (a, b) \mapsto a \vee b$$

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a	b	$a \vee b$

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a	b	$a \vee b$
0	0	0

OR Gate

Function

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0	1	1
1	0	1

OR Gate

Function

$$\vee : B^2 \to B,\ (a, b) \mapsto a \vee b$$

with

a	b	$a \vee b$
0	0	0
0	1	1
1	0	1
1	1	1

XOR Gate (Exclusive Or)

Function

$$\dot{\vee} : B^2 \to B,\ (a, b) \mapsto a \,\dot{\vee}\, b$$

with

XOR Gate (Exclusive Or)

Function

$$\dot{\vee} : B^2 \to B,\ (a, b) \mapsto a \,\dot{\vee}\, b$$

with

a	b	$a \,\dot{\vee}\, b$

XOR Gate (Exclusive Or)

Function

$$\dot{\vee} : B^2 \to B,\ (a, b) \mapsto a \,\dot{\vee}\, b$$

with

a	b	$a \,\dot{\vee}\, b$
0	0	0

XOR Gate (Exclusive Or)

Function

$$\dot{\vee} : B^2 \to B,\ (a, b) \mapsto a \,\dot{\vee}\, b$$

with

a	b	$a \,\dot{\vee}\, b$
0	0	0
0	1	1

XOR Gate (Exclusive Or)

Function

$$\dot{\vee} : B^2 \to B,\ (a, b) \mapsto a \,\dot{\vee}\, b$$

with

a	b	$a \,\dot{\vee}\, b$
0	0	0
0	1	1
1	0	1

XOR Gate (Exclusive Or)

Function

$$\dot{\vee} : B^2 \to B,\ (a, b) \mapsto a \mathbin{\dot{\vee}} b$$

with

a	b	$a \mathbin{\dot{\vee}} b$
0	0	0
0	1	1
1	0	1
1	1	0

Associativity

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

Associativity

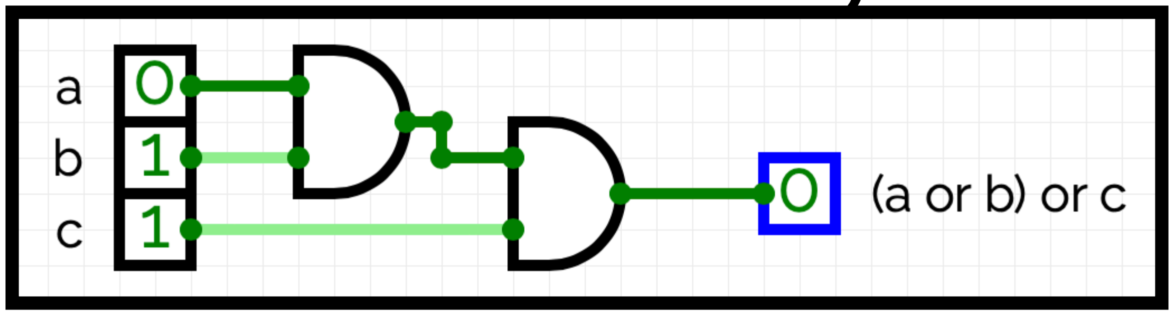
Because of

$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

we can (for example) define

$$a \wedge b \wedge c := (a \wedge b) \wedge c$$

Associativity



$$(a \wedge b) \wedge c = a \wedge (b \wedge c)$$

we can (for example) define

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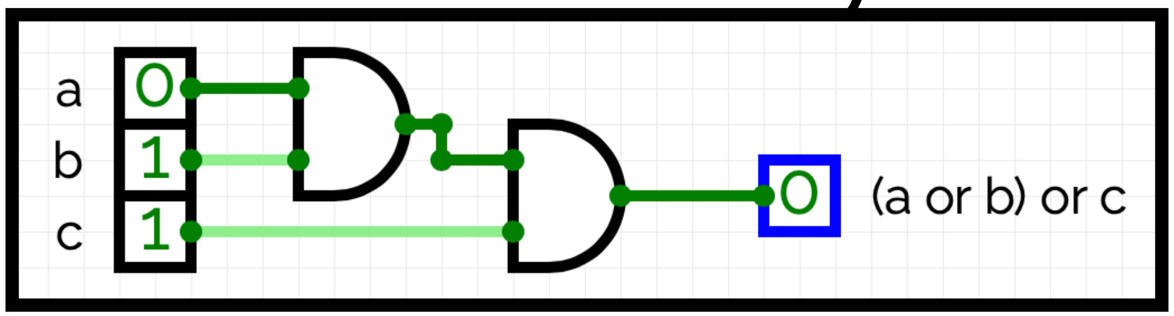
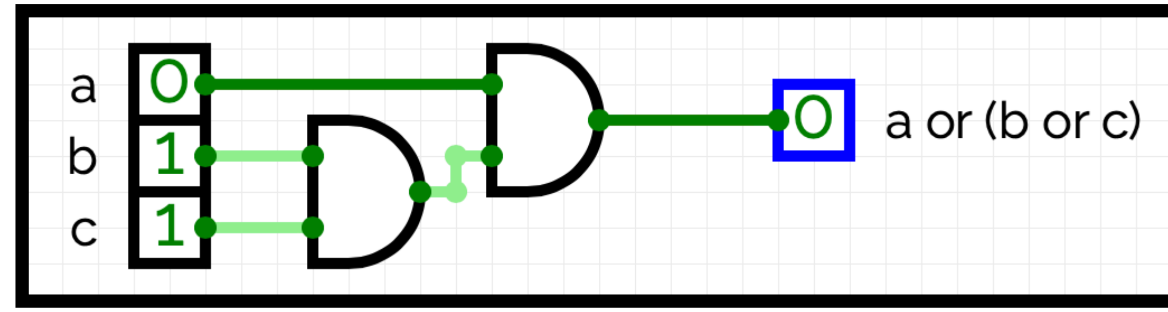
Associativity





$$(a \land b) \land c = a \land (b \land c)$$

we can (for example) define

$$a \land b \land c := (a \land b) \land c$$

Convention: Operator Precedence

Operator	***Precedence***
$\neg$	***4***
$\wedge$	***3***
$\dot{\vee}$	***2***
$\vee$	***1***

$$a \vee (b \wedge c) = a \vee b \wedge c$$

Example

$$\neg a \wedge b \vee a \wedge \neg b$$

Example

$$\neg a \wedge b \vee a \wedge \neg b$$

$$= ((\neg a) \wedge b) \vee (a \wedge (\neg b))$$

a	b	$\neg a$	$\neg b$	$\neg a \wedge b$	$a \wedge \neg b$	$\neg a \wedge b \vee a \wedge \neg b$
0	*0*	*1*	*1*	*0*	*0*	*0*
0	*1*	*1*	*0*	*1*	*0*	*1*
1	*0*	*0*	*1*	*0*	*1*	*1*
1	*1*	*0*	*0*	*0*	*0*	*0*